MATHEMATICAL ANALYSIS OF AN AGE-STRUCTURED MODELLING FOR SOLID WASTE GENERATION MANAGEMENT SYSTEM AND TREATMENT IN MAKINDYE DIVISION OF KAMPALA

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ABSTRACT

Recently, various regulatory policies towards disposal of solid waste generation have not been successful due to improper management of solid waste generation in the communities. This study aimed at assessing the impact of population growth on solid waste generation in Kampala. A deterministic compartmental model is proposed and developed by classifying the population into three age structure and each group has its own rate of solid waste generation and natural death rate using both qualitative and quantitative approaches. The population is assumed to increase due to birth rate and migration rate. Primary data were obtained using a survey questionnaire to 400 respondents in the Makindye Division. The model proposed is analysed qualitatively using the theory of stability differential equations and SPSS software packages. Numerical simulation is employed using Excel Micro Software in order to validate and support the qualitative results. The study found that, both numerical and analytical solutions affirm that the accumulation of solid waste increases with an increase in the fast population growth in the division. The result shows that increase in the solid waste generation treatment effort leads to a decrease in the accumulation of solid waste generation. The results from the data collected revealed that management and environment is significantly correlated with p-value $(p < 0.0005)$. *It is also seen from the results that management and health is significantly correlated with p-value* $(p < 0.002)$. *It is concluded that in order to have an environment free of health hazards, concerted treatment effort is required by all stake holders involved.*

Keywords: *Age-Structured, Endemic Equilibrium, Routh-Hurwitz's criterion, Health hazard*

Introduction

Solid Waste Generation System can be traced back to the olden times when humans started to generate solid waste when they settled down in the beginning of time into small non-nomadic communities at around 10,000 BC Senzige *et al*., (2014). Human activities within households, markets, organizations, and communities which directly or indirectly generate solid waste could be via agricultural, commercial, or domestic households. The solid waste generated from various households are highly heterogeneous in nature and are made up of inevitable solid waste streams namely plastics, food waste, papers, yard waste, metals, glass, textiles, leathers, and miscellaneous materials among others. The consequence of solid waste from fast increase in population, enormous enlargement of the municipal and the life changing, proliferation in population and salary leads to an increase in manufacturing of goods and services and consequently sewages are released into the surroundings. As more people migrate to a municipality, either through birth or migration population increases give on to lofty amount of solid waste generation, thus give rise to harmful consequences to human being and other living creatures [Afon and Okewole (2007), Arena and Di Gregorio (2014). Arena *et al*., (2016)].

Indeed, several studies, in the recent times, have indicated various diseases in Makindye Division due to poor management of solid waste generation such as cholera, flu, diarrhoea and malaria diseases, to mention but just a few. Urban solid waste management is calling for increasing attention, as it can easily be observed that too much garbage is lying uncollected on the streets, causing inconvenience, environmental pollution, and posing a public health risk. In Makindye Division it is assumed that 40 percent of the solid waste generated is being collected daily and the remaining uncollected solid waste goes to the trenches, valleys, and are dumped on the road side, drains, which consequently block the drainage system and lead to floods, insects, vectors and wide spread diseases Nyakaana, (1997). Many of the urban cities are facing large quantities of solid waste generation as well as the challenge of where to dispose the solid waste. Accumulation of solid waste generation in our environment is a natural phenomenon that makes the environment untidy for humans 'health which consequently leads to various societal issues such as environmental, economic and health challenges. Solid waste accumulation system is dynamic in nature coupled with the waste dumped discretely at the dump sites during the day-to-day activities [Ministry of Health Uganda, (2016), Lebersorger & Beigl (2011) and Makinde *et al*., (2014)].

Chuckwu and Ihueze (2017) investigate on mathematical model of solid refuse administration processes. Chira and Taweep (2015) conduct a research on the representations of urban refuse creation and gathering in Thailand. Senzige *et al.*, (2014) investigated on statistical dynamics of refuse creation and cure in the residents' development. Wafula, S.T., Musiime, J. and Oporia, F. (2019). Health care waste management among health workers and associated factors in primary health care facilities in Kampala City of Uganda. Several attempts in modelling solid waste generation management system and treatment have been undertaken by many researchers recently. However, age-structured modelling of solid waste generation management system and treatment has not been undertaken in Makindye Division, this necessitates the study in the Division. This study improved on Nyakaana (1997) by incorporating age-structured and natural death rate for each group. The objectives of the work were to:

i. obtain the endemic equilibrium of the model system;
ii. identify the age-structure that is most susceptible to diseases in the Division;

Materials and Methods

Based on the numerous empirical studies on the solid waste generation, this paper proposed a model that determined the effect of solid waste generation between the juvenile, adult and the senior age-structured. Compartmental deterministic model was employed using Statistical Package for the Social Sciences (SPSS), the theory of stability differential equations via Jacobian matrix and Routh-Hurwitz's criterion. Numerical solutions were obtained through Excel Micro software.

Model Formulation

This study model solid waste generation and treatment based on residents' population dynamics. This study model resident's growth to predict solid waste generation as a direct relationship between inhabitants' growth and solid waste generation rates. It was developed on the basic assumption that solid waste generation rate increases with an increased population growth due to the birth rate and migration rate. The population was grouped into juvenile(age 10-17) denoted as B_j, adult (age 18-49) denoted as B_a and senior (age 50 and above) denoted as B_s, π_h is denoted as the birth rate or recruitment rate into the population, ξ is the migration rate, the natural death rate for the juvenile is denoted by μ_j, the natural death rate for the adult and senior are denoted by μ_a and μ_j respectively. The total population that increases over time is assumed to be N_h

The survival rate of juvenile, adult and senior age group are I_1, I_2 and I_3 respectively. The model system was based on the basic assumption that the rate of solid waste generation increases with an increase in the population. The generation of solid waste rates due to the interaction of the various age-groups with the materials of solid waste generated are assumed as δ_1, δ_2 and δ_3 for the juvenile, adult and senior respectively. In order to keep the environment clean an effort η is assumed and will be employed or applied at the rate of α_1. the effort η in this study implies the assumed measures required or needed by the community and government agencies in order to keep their environment clean. The solid waste generation rates due to natural causes such as tree leaves is given as k. The minimum quantity of solid waste generation is assumed to be Q_H. When effort is applied, the solid waste generation increases at a rate of

β_1 due to the inadequate available resources, the effort decreases at a rate of Λ and the decay natural rate is γ. The ordinary differential equations for the model system are given below:

$$\frac{dB_j}{dt} = \pi_h(1-\xi) - I_1 B_j - \mu_j B_j \tag{1}$$

$$\frac{dB_a}{dt} = I_1 B_j - I_2 B_a - \mu_a B_a \tag{2}$$

$$\frac{dB_s}{dt} = I_2 B_a - I_3 B_s - \mu_s B_s \tag{3}$$

$$\frac{dQ_0}{dt} = \delta_1 B_j + \delta_2 B_a + \delta_3 B_s + k - \gamma Q_0 - \alpha_1 \eta \tag{4}$$

$$\frac{d\eta}{dt} = (Q_0 - Q_H)\beta_1 - \Lambda\eta \tag{5}$$

Sample Size

Following Slovin's formula of sample techniques, a sample size of 400 people was selected from the study total population of 393,008.

$$n = \frac{N}{1+N(e^2)} = \frac{393{,}008}{1+393{,}008(0.0025)} = 399.5932975 \approx 400 \tag{6}$$

Theorem 1: Let D be the closed set given by

$D = \{B_j, B_a, B_s, Q_0, \eta) \in \Re^5_+ : Q_0 \leq Q_0^*,\ \eta \leq \eta^*\}$ are positively invariant and well posed.

Proof:

If $B_j(0), B_a(0), B_s(0), Q_0(0)$, and $\eta(0)$ are all non-negative. Similarly, $B_j(t), B_a(t), B_s(t), Q_0(t)$, and $\eta(t)$ are non-negative for $t > 0$. Additionally,

$\lim_{t\to\infty} Q_0(t) \leq Q_0^*$ and $\lim_{t\to\infty} \eta(t) \leq \eta^*$

when

$$Q_0^* = \frac{\delta_1 B_j + \delta_2 B_a + \delta_3 B_s + k - \alpha_1\eta}{\gamma},\ \eta^* = \frac{(Q_0 - Q_H)\beta_1}{\Lambda} \tag{7}$$

In order to find the endemic equilibrium point, equations (1) -(5) are equated to zero and then solved analytically

$$B_j^* = \frac{\pi_h(1-\xi)}{(I_1-\mu_j)} \quad (8)$$

$$B_a^* = \frac{I_1\pi_h(1-\xi)}{(I_1-\mu_j)(I_2-\mu_a)} \quad (9)$$

$$B_s^* = \frac{I_1\pi_h(1-\xi)I_2}{(I_1-\mu_j)(I_2-\mu_a)(I_3-\mu_s)} \quad (10)$$

$$Q_0^* = \frac{\delta_1 B_j + \delta_2 B_a + \delta_3 B_s + k - \alpha_1\eta}{\gamma} \quad (11)$$

$$\eta^* = \frac{(Q_0 - Q_H)\beta_1}{\Lambda} \quad (12)$$

Stability Analysis of the Model

In order to obtain the stability analysis, let $B_j = x_1, B_a = x_2, B_s = x_3, Q_0 = x_4, \eta = x_5$. Jacobian matrix of the model system is employed and the eigenvalue of the model is obtained as follows:

$$\varepsilon_0 = (B_j^*, B_a^*, B_s^*, Q_0^*, \eta^*)\ ,\ \varepsilon_0 = (x_1, x_2, x_3, x_4, x_5)^T\ ,\ \varepsilon_1 = (B_j^*, B_a^*, B_s^*, 0, 0) \quad (13)$$

$$J(\varepsilon_0) = \begin{bmatrix} -J_{11} & 0 & 0 & 0 & 0 \\ I_1 & -J_{22} & 0 & 0 & 0 \\ 0 & I_2 & -J_{33} & 0 & 0 \\ 0 & 0 & 0 & -J_{44} & -\alpha_1 \\ 0 & 0 & 0 & \beta_1 & -J_{55} \end{bmatrix} \quad (14)$$

where $J_{11} = I_1 + \mu_j + \lambda, J_{22} = I_2 + \mu_a + \lambda, J_{33} = I_3 + \mu_s + \lambda, J_{44} = \gamma + \lambda, J_{55} = \Lambda + \lambda$

$$J(\varepsilon_{01}) = \begin{bmatrix} -J_{11} & 0 & 0 \\ 0 & -J_{44} & -\alpha_1 \\ 0 & \beta_1 & -J_{55} \end{bmatrix},\ (\gamma+\lambda)(\lambda+\Lambda) + \alpha_1\beta_1 = 0 \quad (15)$$

From equation (10) when $Q_0 = \eta = 0$ it yields

$$(I_1 + \mu_j + \lambda)[(I_2 + \mu_a + \lambda)(\lambda + \mu_s)] = 0 \quad (16)$$

Let $\beta_1 = 0.5, \Lambda = 0.89, \gamma = 0.97, \alpha_1 = -0.67, I_2 = 0.94, I_1 = 0.3, \mu_h = 2.673, \pi_h = 37.777$

$$H = e^{-2t}\left(A \sin \sqrt{2}t + B \cos \sqrt{2}t\right) \tag{17}$$

Routh-Hurwitz's criterion [12]

From equation (14)

$$\lambda^2 + \lambda(\gamma + \Lambda) + \gamma\Lambda + \beta_1 \alpha_1 = 0, \quad a_2\lambda^2 + a_1\lambda^1 + a_0\lambda^0 = 0 \tag{18}$$

$$a_1 > 0, \quad a_0 > 0, (\gamma + \Lambda) > 0 \text{ and } \gamma\Lambda + \alpha_1\beta_1 > 0$$

where $R_1 = -\dfrac{\gamma}{\Lambda}$ and $R_2 = -\dfrac{\alpha_1\beta_1}{\gamma\Lambda}$, $a_1 = \gamma + \Lambda$, $a_0 = \gamma\Lambda + \alpha_1\beta_1$

From equation (16)

$$A_3\lambda^3 + A_2\lambda^2 + A_1\lambda^1 + A_0\lambda^0 = 0$$

where $A_0 = \mu_s(I_1 + \mu_j)(I_s + \mu_a)$, $A_1 = (I_1 + 1)(I_2 + \mu_a + \mu_s) + \mu_s(I_s + \mu_a)$,
$A_3 = 1$, $A_2 = I_1 + \mu_a + \mu_s + \mu_j + I_2$

$$\lambda^n = \begin{bmatrix} B_1 & B_3 & B_5 & B_7 \\ b_1 & b_2 & b_3 & b_4 \\ c_1 & c_2 & c_3 & c_4 \\ d_1 & d_2 & d_3 & d_4 \end{bmatrix} = \begin{bmatrix} B_0 & B_2 & 0 & 0 \\ B_1 & 1 & 0 & 0 \\ -0.102 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$ where $n = 0, 1, 2, 3$.

Lemma 3.2: The solid waste generation endemic equilibrium ε_0 is locally asymptotically stable (LAS) and ε_1 is endemically unstable.

Proof: It is obvious from the consequences of the Jacobian matrix that the trivial solution always exists and the local stability of the model system is achieved which implies that the endemic equilibrium is locally asymptotically stable. Following Routh-Hurwitz's criterion [12] $R_1 < 1, R_2 < 1$ and ε_1 is unstable. It implies that in the absence of the effort applied, all the solid waste generated naturally deteriorate to zero. Consequently, the treatment strategy was always the most cost effective. This completes the proof.

Figure 1: Map of Makindye Division, Kampala.

Source: (Excerpt from www. Map data, March, 2020)

Results and Discussion

Presentation of Results

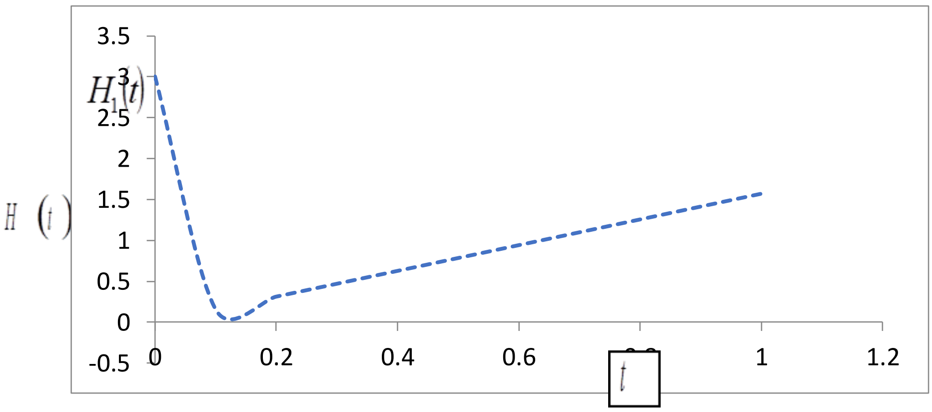

Figure 2: *The graph of population growth with various values of* $I_1 = 0.1, 0.2, 0.5,\ I_2 = 0.4, 0.6, 1, \mu_j = 2.8809,\ \mu_h = 2.673,\ \pi_h = 37.777$

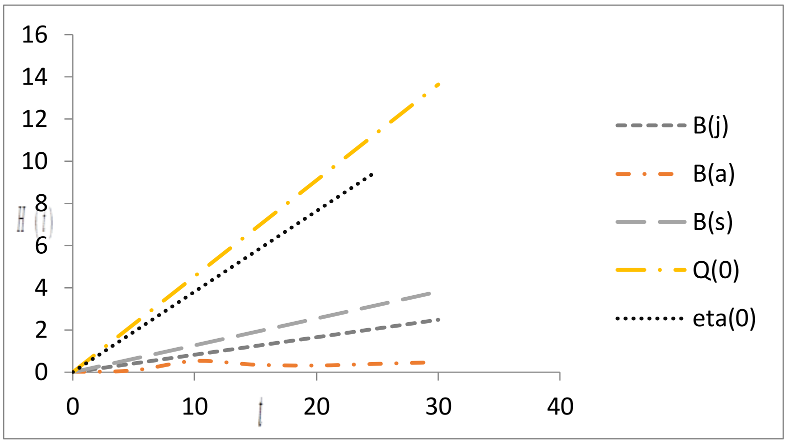

Figure 3: The graph of population growth with treatment for various values of $I_1 = 0.1, 0.2, 0.5,\ \ I_2 = 0.4, 0.6, 1, \mu_j = 2.8809,\ \mu_h = 2.673,\ \pi_h = 37.777$

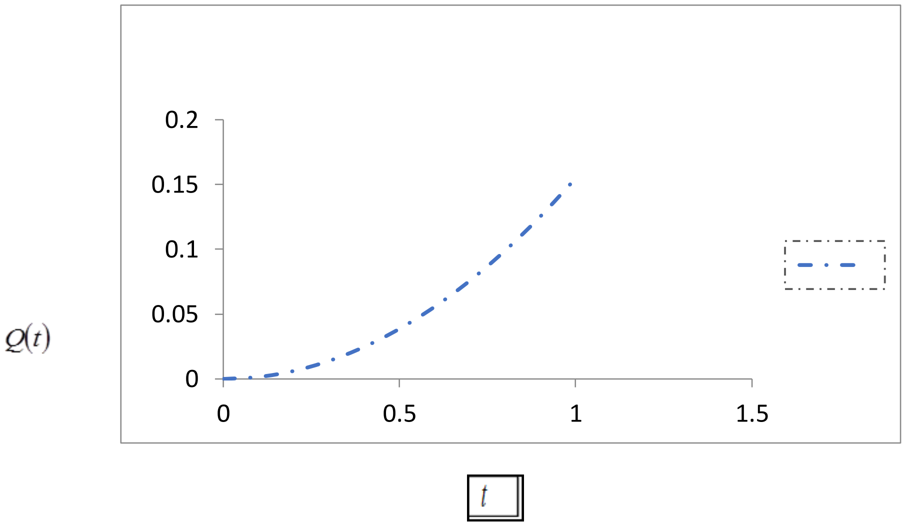

Figure 4: *The graph of effort treatment with various values of* $I_1 = 0.1, 0.2, 0.5,\ \ I_2 = 0.4, 0.6, 1, \mu_c = 2.8809,\ \mu_h = 2.673,\ \alpha = 37.777$

Table 1: Residential's Correlation on Environment, Management and Health

		HEALTH	ENVIRONMENT	MANAGEMENT OF SOLID WASTE
HEALTH	Pearson Correlation	1	.447*	.311**
	Sig. (2-tailed)		.000	.002
	N	399	399	399
ENVIRONMENT	Pearson Correlation	.447**	1	.611**
	Sig. (2-tailed)	.000		.000
	N	399	399	399
MANAGEMENT OFSOLIDWASTE	Pearson Correlation	.311**	.611**	1
	Sig. (2-tailed)	.002	.000	
	N	399	399	399

*. Correlation is significant at the 0.05 level (2-tailed).
Correlation is significant at the 0.01 level (2-tailed).

Table 2: Classification of Residentials' Age

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	0-9	95	23.75	23.75	23.75
	10-17	105	26.25	26.25	50.0
	18-49	150	37.5	37.5	87.5
	50 & above	50	12.5	12.5	100.0
	Total	400	100.0	100.0	

Source: Analysis of respondents from SPSS

Discussion of Findings

This study assesses the impact of population growth on solid waste generation as it differs from location to location in connection to population magnitude within Makindye municipality. Since the eigenvalues in equation (14) are all negative, it implies that solid waste endemic equilibrium is achieved and ε_1 is endemically unstable. It is noted from equation (15) that the model system is stable. Nevertheless, a magnitude quantity Q_H is assumed and allowed to be exceeded by the agencies in the district above which the effort η must be applied. It implies that ε_1 is the assumed point whereby the solid waste generation is moving in different direction everywhere thereby causing serious harm, injury or danger to the health of the people and a source of environmental pollution to the community. Epidemiologically, Lemma 2 implies that solid waste accumulation can be eliminated from the community when $R_1 < 1$ and $R_2 < 1$. On the other hand, the ability of the treatment is increased without bound in collaboration with other intervention methods taken by all stake holders involved within Kampala.

The result from Figure 2 shows that solid waste is enhanced with an increase in the growth of the population as result of increased immigrants, which consequently leads to increase in the rate of solid waste generation. The result from Figures 3-4 show that increases in the effort treatment reduces the accumulation of solid waste in the environment. It implies that increase in the treatment effort leads to a decrease in the accumulation of the solid waste. Also, the rate of change in the treatment effort leads to decrease in the accumulation of solid waste. The result from table 1 shows that management and environment is significantly correlated with p-value $(p < 0.0005)$. It is also seen that management and health is significantly correlated with p-value $(p < 0.002)$. Consequently, it is deduced that environment and management is significantly correlated with p-value $(p < 0.0005)$ which implies that uncollected waste leads to health hazards in this division. Similarly, the result shows that environment and health is sig9nificantly correlated with p-value $(p < 0.0005)$ which implies that the environment is faced with various diseases associated with solid waste such as breeding sites of mosquitoes (which consequently leads to malaria transmission), diarrhoea (which leads to an increase in the death rate of children under 5 years in the district), cholera, cough and flu is in agreement with [Ministry of Health, Uganda (2010) and UBOS & ICF. (2011)].

Table 2 shows that age group 18-49 (adult age-group) is the age-structure that deals with solid waste generation. It is the age group in which the immigrant rate is high which implies increase in the immigrant level consequently leads to population increase in urban city. It is noted that age groups 0-9, 10-17 and 18-49 are the most susceptible to malaria and diarrhea [Ministry of Health Uganda, (2016), UBOS (2001), Ministry of Health. (2014) and Wafula, *et al*., (2019)].

Conclusions and Recommendations

The study assesses the impact of population growth on solid waste creation as it differs from location to location in connection to population magnitude within Makindye municipality. Findings from results also revealed that there is a corresponding correlation between population and the aggregate of solid waste generation. The result also shows that there is disparity in the quantity of solid waste generation between high and low salary locality. It is concluded that concerted treatment effort is required by all stake holders involved and there is need for the application of sustainable environmental education to have an environment free of health hazards. Based on the findings of this studies, this research studies have emphasized issues of policy applicability as follows:

i. There is need for more awareness campaigns and programs for proper solid waste management and generation.
ii. There should be a concerted responsibility from both government and non-governmental organizations in raising the households' levels of perception and cognition approach towards solid waste management and generation system in the sampled Division in Kampala.
iii. Treatment of the solid waste generation should be carried out regularly in order to eliminate to the barest minimum the health hazard determined in the community.

Implications of the Study

Based on the findings of this study it is suggested that concerted effort regarding the treatment of accumulation of solid waste is required by all stakeholders involved in the management of solid waste in order to reduce environmental pollution.

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